

Refining the Unruh metric tensor uncertainty principle via Schwarzschild Geometry and Planckian Space-time with initial nonzero entropy

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Abstract

First, we restate a proof of a highly localized special case of a metric tensor uncertainty principle first written up by Unruh. Unruh did not use the Robertson-Walker geometry which we do, and it so happens that the dominant metric tensor we will be examining, is variation in δg_{tt} . The metric tensor variations given by δg_{rr} , $\delta g_{\theta\theta}$ and $\delta g_{\phi\phi}$ are negligible, as compared to the variation δg_{tt} . Afterwards, what is referred to by Barbour as emergent duration of time δt is from the Heisenberg Uncertainty principle (HUP) applied to δg_{tt} in such a way as to give, in the Planckian space-time regime a nonzero minimum non zero lower ground to a massive graviton, $m_{graviton}$. The lower bound to the massive graviton, is influenced by δg_{tt} and kinetic energy which is in the Planckian emergent duration of time δt as $(E - V)$.

Key words: Massive Gravitons, Heisenberg Uncertainty Principle (HUP), Riemannian-Penrose Inequality

i. Introduction

The first matter of business will be to introduce a framework of the speed of gravitons in “heavy gravity”. Heavy Gravity is the situation where a graviton has a small rest mass and is not a zero-mass particle, and this existence of “heavy gravity” is important since eventually, as illustrated by Will [1,2] gravitons having a small mass could possibly be observed via their macroscopic effects upon astrophysical events. Secondly, our manuscript’s inquiry also will involve an upper bound to the rest mass of a graviton. The second aspect of the inquiry of our

manuscript will be to come up with a variant of the Heisenberg Uncertainty principle (HUP), involving a metric tensor, as well as the Stress energy tensor, which will in time allow us to establish a lower bound to the mass of a graviton, preferably at the start of cosmological evolution.

We reference what was done by Will in his living reviews of relativity article as to the ‘Confrontation between GR and experiment’. Specifically, we make use of his experimentally based formula of [1, 2], with $v_{graviton}$ the speed of a graviton, and $m_{graviton}$ the rest mass of a graviton, and $E_{graviton}$ in the inertial rest frame given as:

$$\left(\frac{v_{graviton}}{c}\right)^2 = 1 - \frac{m_{graviton}^2 c^4}{E_{graviton}^2} \quad (1)$$

Furthermore, using [2], if the rest mass of a graviton is very small, we can make a clear statement of

$$\begin{aligned} \frac{v_{graviton}}{c} &= 1 - 5 \times 10^{-17} \cdot \left(\frac{200Mpc}{D}\right) \cdot \left(\frac{\Delta t}{1sec}\right) \\ B &1 - 5 \times 10^{-17} \cdot \left(\frac{200Mpc}{D}\right) \cdot \left(\frac{\Delta t = \Delta t_a - (1+z) \cdot \Delta t_b}{1sec}\right) \\ \Leftrightarrow \frac{2m_{graviton} c^2}{E_{graviton}} &\approx 5 \times 10^{-17} \cdot \left(\frac{200Mpc}{D}\right) \cdot \left(\frac{\Delta t_a - (1+z) \cdot \Delta t_b}{1sec}\right) \end{aligned} \quad (2)$$

Here, Δt_a is the difference in arrival time, and Δt_e is the difference in emission time/in the case of the early Universe, i.e. near the big bang, then if in the beginning of time, one has, if we assume that there is an average $E_{graviton} \approx h \cdot \omega_{graviton}$, and

$$\begin{aligned} \Delta t_a &\sim 4.3 \times 10^{17} \text{ sec} \\ \Delta t_e &\sim 10^{-33} \text{ sec} \\ z &\sim 10^{50} \end{aligned} \quad (3)$$

Then, $\left(\frac{\Delta t_a - (1+z) \cdot \Delta t_b}{1sec}\right) \sim 1$ and if $D \sim 4.6 \times 10^{26} \text{ meters} = \text{radii}(\text{universe})$, so one can set

$$\left(\frac{200Mpc}{D}\right) \sim 10^{-2} \quad (4)$$

And if one sets the mass of a graviton [3] into Eq. (1), then we have in the present era, that if we look at primordial time generated gravitons, that if one uses the

$$\begin{aligned}
\Delta t_a &\sim 4.3 \times 10^{17} \text{ sec} \\
\Delta t_e &\sim 10^{-33} \text{ sec} \\
z &\sim 10^{55}
\end{aligned} \tag{5}$$

Note that the above frequency, for the graviton is for the present era, but that it starts assuming genesis from an initial inflationary starting point which is not a space – time singularity.

Note this comes from a scale factor, if $z \sim 10^{55} \Leftrightarrow a_{\text{scale-factor}} \sim 10^{-55}$, i.e. 55 orders of magnitude smaller than what would normally consider, but here note that the scale factor is not zero, so we do not have a space – time singularity.

We will next discuss the implications of this point in the next section, of a non-zero smallest scale factor. Secondly the fact we are working with a massive graviton, as given will be given some credence as to when we obtain a lower bound, as will come up in our derivation of

$$\begin{aligned}
&\left\langle (\delta g_{uv})^2 (\hat{T}_{uv})^2 \right\rangle \geq \frac{\hbar^2}{V_{\text{Volume}}^2} \\
\text{modification of the values[3]} &\xrightarrow{uv \rightarrow tt} \left\langle (\delta g_{tt})^2 (\hat{T}_{tt})^2 \right\rangle \geq \frac{\hbar^2}{V_{\text{Volume}}^2} \\
&\& \delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^+
\end{aligned} \tag{6}$$

2. Nonzero scale factor, initially and what this is telling us physically. Starting with a configuration from Unruh.

Begin with the starting point of [4, 5]

$$\Delta l \cdot \Delta p \geq \frac{\hbar}{2} \tag{7}$$

We will be using the approximation given by Unruh [4, 5],

$$\begin{aligned}
(\Delta l)_{ij} &= \frac{\delta g_{ij}}{g_{ij}} \cdot \frac{l}{2} \\
(\Delta p)_{ij} &= \Delta T_{ij} \cdot \delta t \cdot \Delta A
\end{aligned} \tag{8}$$

If we use the following, from the Robertson-Walker metric [6].

$$\begin{aligned}
g_{tt} &= 1 \\
g_{rr} &= \frac{-a^2(t)}{1-k \cdot r^2} \\
g_{\theta\theta} &= -a^2(t) \cdot r^2 \\
g_{\phi\phi} &= -a^2(t) \cdot \sin^2 \theta \cdot d\phi^2
\end{aligned} \tag{9}$$

Following Unruh [4, 5], write then, an uncertainty of metric tensor as, with the following inputs.

$$a^2(t) \sim 10^{-110}, r \equiv l_p \sim 10^{-35} \text{ meters} \tag{10}$$

Then, the surviving version of Eq. (7) and Eq. (8) is, then, if $\Delta T_{tt} \sim \Delta\rho$

$$\begin{aligned}
V^{(4)} &= \delta t \cdot \Delta A \cdot r \\
\delta g_{tt} \cdot \Delta T_{tt} \cdot \delta t \cdot \Delta A \cdot \frac{r}{2} &\geq \frac{\hbar}{2} \\
\Leftrightarrow \delta g_{tt} \cdot \Delta T_{tt} &\geq \frac{\hbar}{V^{(4)}}
\end{aligned} \tag{11}$$

This Eq. (11) is such that we can extract, up to a point the HUP principle for uncertainty in time and energy, with one very large caveat added, namely if we use the fluid approximation of space-time[6] for the stress energy tensor as given in Eq. (12) below.

$$T_{ii} = \text{diag}(\rho, -p, -p, -p) \tag{12}$$

Then

$$\Delta T_{tt} \sim \Delta\rho \sim \frac{\Delta E}{V^{(3)}} \tag{13}$$

Then, Eq. (11) and Eq. (12) and Eq. (13) together yield

$$\delta t \Delta E \geq \frac{\hbar}{\delta g_{tt}} \neq \frac{\hbar}{2} \tag{14}$$

$$\text{Unless } \delta g_{tt} \sim O(1)$$

How likely is $\delta g_{tt} \sim O(1)$? Not going to happen. Why? The homogeneity of the early universe will keep

$$\delta g_{tt} \neq g_{tt} = 1 \quad (15)$$

In fact, we have that from Giovannini [6], that if ϕ is a scalar function, and $a^2(t) \sim 10^{-110}$, then if

$$\delta g_{tt} \sim a^2(t) \cdot \phi \ll 1 \quad (16)$$

Then, there is no way that Eq. (14) is going to come close to $\delta t \Delta E \geq \frac{\hbar}{2}$. Hence, the Mukhanov suggestion as will be discussed toward the end of this article, is not feasible. Finally, we will discuss a lower bound to the mass of the graviton.

3. How we can justify writing very small $\delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^+$ values?

To begin this process, we will break it down into the following coordinates.

In the $rr, \theta\theta$ and $\phi\phi$ coordinates, we will use the Fluid approximation, $T_{ii} = \text{diag}(\rho, -p, -p, -p)$ [7] with

$$\begin{aligned} \delta g_{rr} T_{rr} &\geq - \left| \frac{\hbar \cdot a^2(t) \cdot r^2}{V^{(4)}} \right| \xrightarrow{a \rightarrow 0} 0 \\ \delta g_{\theta\theta} T_{\theta\theta} &\geq - \left| \frac{\hbar \cdot a^2(t)}{V^{(4)} (1 - k \cdot r^2)} \right| \xrightarrow{a \rightarrow 0} 0 \\ \delta g_{\phi\phi} T_{\phi\phi} &\geq - \left| \frac{\hbar \cdot a^2(t) \cdot \sin^2 \theta \cdot d\phi^2}{V^{(4)}} \right| \xrightarrow{a \rightarrow 0} 0 \end{aligned} \quad (17)$$

If as an example, we have negative pressure, with $T_{rr}, T_{\theta\theta}$ and $T_{\phi\phi} < 0$, and $p = -\rho$, then the

only choice we have, then is to set $\delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^+$, since there is no way that

$p = -\rho$ is zero valued.

Having said this, the value of δg_{tt} being nonzero, will be part of how we will be looking at a lower bound to the graviton mass which is not zero.

4. Lower bound to the graviton mass using Barbour's emergent time

To start this approximation, we will be using Barbour's value of emergent time [8, 9] restricted to the Plank spatial interval and massive gravitons, with a massive graviton [10]

$$(\delta t)_{emergent}^2 = \frac{\sum_i m_i l_i \cdot l_i}{2 \cdot (E - V)} \rightarrow \frac{m_{graviton} l_P \cdot l_P}{2 \cdot (E - V)} \quad (18)$$

Initially, as postulated by Babour [8, 9], this set of masses, given in the emergent time structure could be for say the planetary masses of each contribution of the solar system. Our identification is to have an initial mass value, at the start of creation, for an individual graviton.

If $(\delta t)_{emergent}^2 = \delta t^2$ in Eq. (11), using Eq. (11) and Eq. (18) we can arrive at the identification of

$$m_{graviton} \geq \frac{2h^2}{(\delta g_{tt})^2 l_P^2} \cdot \frac{(E - V)}{\Delta T_{tt}^2} \quad (19)$$

Key to Eq. (19) will be identification of the kinetic energy which is written as $E - V$. This identification will be the key point raised in this manuscript. Note that [11] raises the distinct possibility of an initial state, just before the 'big bang' of a kinetic energy dominated 'pre inflationary' universe. i.e., in terms of an inflation $\phi \gg (P.E \sim V)$ [7]. The key finding which is in [11] is, that, if the kinetic energy is dominated by 'inflation' that,

$$K.E. \sim (E - V) \sim \phi \propto a^{-6} \quad (20)$$

This is done with the proviso that $w < -1$, in effect, what we are saying is that during the period of the 'Planckian regime' we can seriously consider an initial density proportional to Kinetic energy, and call this K.E. as proportional to [7]

$$\rho_w \propto a^{-3(1-w)} \quad (21)$$

If we are where we are in a very small Planckian regime of space-time, we could, then say write Eq.

(21) as proportional to $g^* T^4$ [7], with g^* initial degrees of freedom, and T the initial temperature as

just before the onset of inflation. The question to ask, what is the value of the initial degrees?

of freedom, and what is the temperature, T, at the start of expansion? For what it is worth, the

supposition, is that there would then be a likelihood for an initial low temperature regime

5. Multiverse and answering the Mukhanov hypothesis. Influence of the Einstein spaces

Here, the initial $a_0 \sim a_{initial} \sim 10^{-55}$, or so and so the density in Eq. (21) at Planck time would, be proportional to the Planck Frequency [7]

$$\omega_p = \frac{1}{t_p} = \sqrt{\frac{c^5}{hG}} = 1.85487 \times 10^{43} \text{ s}^{-1} \sim 1.85 \times 10^{43} \text{ Hz} \quad (22)$$

This is at the instant of Planck time. We can then ask what an initial time contribution before the onset of Planck time would be. Does Eq. (22) represent the initial value of graviton frequency?

This value of the frequency of a graviton, which would be red shifted enormously would be in tandem with an initial time step of as given by [12]

$$t_{initial} \approx \frac{1}{\sqrt{6\pi\rho_{initial}}} \sim \frac{a_0^2}{\sqrt{6\pi}} \quad (23)$$

This value for the initial time step would be probably lead to Pre Planckian time , i.e. smaller than 10^{-43} seconds, which then leads us to consider, what would happen if a multi verse contributed to initial space-time conditions as seen in Eq. (11) above. If the cosmic fluid approximation as given by Eq. (12) were legitimate, and one could also look at Eq. (13),

$$m_{graviton} \geq \frac{2h^2}{(\delta g_{tt})^2 l_P^2} \cdot \frac{(E-V)}{\Delta T_{tt}^2} \Rightarrow (\delta g_{tt})^2 \geq \frac{2h^2}{m_{graviton} \cdot l_P^2} \cdot \frac{(E-V)}{\Delta T_{tt}^2} \quad (24)$$

Then if one is looking at a multiverse, we first will start at the Penrose hypothesis for a cyclic conformal universe, starting with [13]

$$\hat{g}^{uv} = \Omega_w g^{uv}$$

$$\Omega_w (new-universe) = (\Omega_w^{-1} old-universe) \quad (25)$$

i.e.

$$\Omega_w \rightarrow \Omega_w^{-1} (inversion)$$

However, in the multiverse contribution to Eq. (12) above, we would have, that

$$\Omega_w^{-1} old-universe \rightarrow \frac{1}{N} \sum_{j=1}^N [\Omega_w^{-1} (inversion)]_j \quad (26)$$

So, does something like this hold? In a general sense?

$$\begin{aligned}
& (\delta g_{uv})^2 \Big|_{\text{initial}} \hat{g}^{uv} = \Omega_{uv} g^{uv} \\
& \rightarrow \frac{1}{N} \sum_{j=1}^N \left[\Omega_{uv}^{-1}(\text{inversion}) \right]_j g^{uv} \\
& \sim 1 / \left[\beta M_{\text{Planck}}^2 \right] + \varepsilon^+
\end{aligned} \tag{27}$$

If the fluid approximation as given in Eq. (12) and Eq. (13) hold, then Eq. (27) conceivably could be identifiable as linkable to.

$$\begin{aligned}
\delta t \Delta E & \geq \frac{\hbar}{(\delta g_{tt})} \equiv \frac{N \cdot \hbar}{\left(\left(\sum_j [\Omega_{tt}^{-1}]_j \right) \cdot \delta g_{tt} \right)} \\
\Leftrightarrow \delta t & \geq \frac{1}{\Delta E} \cdot \frac{N \cdot \hbar}{\left(\left(\sum_j [\Omega_{tt}^{-1}]_j \right) \cdot \delta g_{tt} \right)} \approx \frac{m_{\text{graviton}} l_P \cdot l_P}{2 \cdot (E - V)}
\end{aligned} \tag{28}$$

If we could write, say

$$\begin{aligned}
\frac{1}{\Delta E} \cdot \frac{N \cdot \hbar}{\left(\left(\sum_j [\Omega_{tt}^{-1}]_j \right) \cdot \delta g_{tt} \right)} & = \frac{N \cdot \hbar / \Delta E}{\left(\sum_j [\Omega_{tt}^{-1} \cdot (\delta g_{tt})]_j \right)} \sim \frac{m_{\text{graviton}} l_P \cdot l_P}{2 \cdot (E - V)} \\
\Leftrightarrow \Delta E & \sim 2 \cdot (E - V)
\end{aligned} \tag{29}$$

&

$$m_{\text{graviton}} l_P \cdot l_P \sim \frac{N \cdot \hbar}{\left(\sum_j [\Omega_{tt}^{-1} \cdot (\delta g_{tt})]_j \right)}$$

Then, if each j is the j th contribution of N “multiverse” contributions to a new single universe being nucleated, one could say that there was, indeed, likely an “averaging” and that the causal barrier which Mukhanov spoke of, as to each δt , and actually to each graviton entering into the present universe, one could mathematically average out the results of a sum up of each of the contributions from each prior to a present universe, according to

$$\frac{N \cdot \hbar}{\left(\sum_j [\Omega_{tt}^{-1} \cdot (\delta g_{tt})]_j \right)} \equiv \frac{\hbar}{\frac{1}{N} \cdot \sum_j [\Omega_{tt}^{-1} \cdot (\delta g_{tt})]_j} \tag{30}$$

If Eq. (30) held, then we could then write

$$\delta t \geq \frac{1}{\Delta E} \cdot \frac{\hbar}{\left(\frac{1}{N} \sum_j [\Omega_{tt}^{-1} \cdot \delta g_{tt}]_j \right)} \approx \frac{m_{\text{graviton}} l_P \cdot l_P}{2 \cdot (E - V)} \tag{31}$$

Instead, we have Eq. (28), and it is safe to say that for each collapsing universe which might contribute to a re cycled universe that the following inequality is significant.

$$\frac{1}{N} \cdot \sum_j [\Omega_{tt}^{-1} \cdot (\delta g_{tt})]_j \neq \frac{1}{N} \cdot \sum_j [\Omega_{tt}^{-1}]_j \cdot \delta g_{tt} \quad (32)$$

Hence, the absence of an averaging procedure, due to a multiverse, would then rule against a causal barrier, as was maintained by Mukhanov, in his discussion with the author, in Marcel Grossman 14, in Italy. Then the possible approximation say of

$$(T_{tt})^2 \sim \omega_{graviton}^2 \propto \beta M_{Planck}^2 \sim \left(t_{initial} \approx \frac{1}{\sqrt{6\pi\rho_{initial}}} \sim \frac{a_0^2}{\sqrt{6\pi}} \right)^{-2} \quad (33)$$

Would not hold, and that may lead to a breakdown of the Causal barrier hypothesis of Mukhanov, which the author emphatically disagreed with.

6. Conclusion. Considering Eq. (6) and Eq. (11) in lieu of Einstein space, and further research questions

A way of solidifying the approach given here, in terms of early universe GR theory is to refer to Einstein spaces, via [14] as well as to make certain of the Stress energy tensor [15] as we can write it as a modified Einstein field equation. With, then \aleph as a constant.

$$R_{ij} = \aleph g_{ij} \quad (34)$$

Here, the term in the left-hand side of the metric tensor is a constant, so then if we write, with R also a constant [15]

$$T_{ij} = -\frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{ij}} = -\frac{1}{8\pi} \cdot [\aleph - R + \Lambda] \cdot g_{ij} \quad (35)$$

The terms, if we use the fluid approximation given by Eq. (12) as well as the metric given in Eq. (9) will then tend to a constant energy term on the RHS of Eq. (35) as well as restricting i, and j, to t and t

So as to recover, via the Einstein spaces, the seemingly heuristic argument given above. Furthermore when we refer to the Kinetic energy space as an inflation where we assume that the potential energy is proportional to V, so as to allow us to write $\aleph \gg (P.E \sim V)$ [7], we can also then utilize the following operator equation for the generation of an ‘inflation field’ given by the following set of equations

$$\begin{aligned}
\phi(t) &= \cos(t\sqrt{K})f + \frac{\sin(t\sqrt{K})}{\sqrt{K}}g \\
f(x) &= \phi(0, x) \\
g(x) &= \frac{\partial\phi(0, x)}{\partial t} \\
-\frac{\partial^2\phi}{\partial t^2} &= K\phi
\end{aligned} \tag{36}$$

In the case of the general elliptic operator K if we are using the Fulling reference, [16] in the case of the above Robertson-Walker metric, with the results that the elliptic operator, in this case become,

$$\begin{aligned}
K &= -\nabla^2 + (m^2 + \xi R) \\
&= -\sum_{i,j} \frac{\partial_i \left(g^{ij} \sqrt{|\det g|} \partial_j \right)}{\sqrt{|\det g|}} + (m^2 + \xi R) \\
&\xrightarrow{i,j \rightarrow t,t} -\frac{\partial^2}{\partial t^2} + (m^2 + \xi R)
\end{aligned} \tag{37}$$

Then, according to [16], if R above, in Eq. (37) is initially a constant, we will see then, if m is the inflation mass,

$$\begin{aligned}
\phi(t) &= \cos(t\sqrt{K})f \\
-\frac{\partial^2}{\partial t^2} &\rightarrow \omega^2 \\
\Leftrightarrow \phi(t) &= \cos(t\sqrt{\omega^2 + (m^2 + \xi R)})
\end{aligned} \tag{38}$$

Then c_1 as an unspecified, for now constant will lead to a first approximation of a Kinetic energy dominated initial configuration, with details to be gleaned from [16,17,18] to give more details to the following equation, R here is linked to curvature of space-time, and m is an inflation mass, connected with the field $\phi(t) = \cos(t\sqrt{K})f$ with the result,

$$\mathcal{E}(t) \approx [\omega^2 + (m^2 + \xi R)] \cdot c_1 \gg V(\phi) \tag{39}$$

If the frequency, of say, Gravitons is of the order of Planck frequency as in Eq. (22), then this term, would likely dominate Eq. (39). More of the details of this will be worked out, and also candidates for the $V(\phi)$ will be ascertained, most likely, we will be looking the Rindler Vacuum as specified in [19] as well as also details of what is relevant to maintain local covariance in the initial space-time fields as given in [20]

Why is a refinement of Eq. (39) necessary?

The details of the elliptic operator K will be gleaned from [16, 17, 18] whereas the details of inflation $\phi \gg (P.E \sim V)$ [7] are important to get a refinement on the lower mass of the graviton as given by the left-hand side of Eq. (24). We hope to do this in the coming year. The mass, \mathbf{m} , in Eq. (37) for inflation, not Graviton, to have links to the beginning of the expansion of the universe. We look to what Corda did, in [21] for guidance as to picking values of \mathbf{m} relevant to early universe conditions.

Finally, as far as Eq. (39) is concerned, there is one serious linkage issue to classical and quantum mechanics, which should be the bridge between classical and quantum regimes, as far as space time applicability. Namely, from Wald (19), if we look at first of all arbitrary operators, A and B

$$(\Delta A)^2 \cdot (\Delta B)^2 \geq \left(\frac{1}{2i} \langle [A, B] \rangle \right) \quad (40)$$

As we can anticipate, the Pre Planckian regime may be the place to use classical mechanics, and then to bridge that to the Planckian regime, which would be quantum mechanical. Taking [19] again, this would lead to a simplistic structure via the following modification of the Hamilton equations of motion, namely we will from (19) get the following re write,

$$\begin{aligned} \frac{dq_\mu}{dt} &= \frac{\partial H}{\partial p_\mu}, \quad \frac{dp_\mu}{dt} = -\frac{\partial H}{\partial q_\mu} \\ H &= H(q_1, \dots, q_n; p_1, \dots, p_n) \\ y &= (q_1, \dots, q_n; p_1, \dots, p_n) \\ \Omega^{\mu\nu} &= 1, \text{ if } \nu = \mu + n \\ \Omega^{\mu\nu} &= 0, \text{ otherwise} \\ \frac{dy^\mu}{dt} &= \sum_{\nu=1}^n \Omega^{\mu\nu} \frac{\partial H}{\partial y^\nu} \end{aligned} \quad (41)$$

Then there exists a re formulation of the Poisson brackets, as seen by

$$\{f, g\} = \Omega^{\mu\nu} \nabla_\mu f \nabla_\nu g \quad (42)$$

So, then the following, for classical observables, f , and g , we could write, by [19]

$$\begin{aligned} \wedge : \Theta &\rightarrow \hat{\Theta} \\ \Theta &= \text{classical} - \text{observable} \\ \hat{\Theta} &= \text{quantum} - \text{observable} \\ \hbar &= 1 \\ [\hat{f}, \hat{g}] &= i \cdot \{f, g\} \end{aligned} \quad (43)$$

Then, we could write, say Eq. (40) and Eq. (43) as

$$\begin{aligned}
[\hat{f}, \hat{g}] &= i \cdot (\{f, g\}) \\
f &= \text{classical} - \text{observable} \\
\hat{f} &= \text{quantum} - \text{observable} \\
(\Delta\hat{f})^2 \cdot (\Delta\hat{g})^2 &\geq \left(\frac{1}{2i} (\langle [\hat{f}, \hat{g}] \rangle) \right) = \left(\frac{1}{2} (\langle \{f, g\} \rangle) \right)
\end{aligned} \tag{44}$$

If so, then we can set, in the interconnection between the Planck regime, and just before the Planck regime, say, by setting classical variables, as given by

$$\begin{aligned}
f &= -\frac{[\aleph - R + \Lambda] \cdot g_{tt}}{8\pi} \\
g &= \delta g_{tt} \\
\end{aligned} \tag{45}$$

Then by utilization of Eq. (44) we may be able to effect more precision in our early universe derivation, especially making use of derivational work, in addition as to what is given here, as to understand how to construct a very early universe partition function Z based upon the inter relationship between Eq. (44) and Eq. (45) so as to write up an entropy based upon, as given in [19]

$$S(\text{entropy}) = \ln Z + \beta E \tag{46}$$

If this program were affected, with a first principle construction of a partition function, we may be able to answer if Entropy were zero in the Planck regime, or something else, which would give us more motivation to examine the sort of partition functions as stated in [22, 23]. See **appendix A** as to possible scenarios. Here keep in mind that in the Planck regime we have nonstandard physics. Appendix A indicates that due to the variation we have worked out in the Planckian regime of space-time that the initial entropy is not zero.

Appendix A, scenarios as to the value of entropy in the beginning of space-time nucleation

We will be looking at inputs from page 290 of [23] so that if $E \sim M \sim \Delta T_{tt} \cdot \delta t_{time} \cdot \Delta A \cdot l_p$

$$S(\text{entropy}) = \ln Z + \frac{(E \sim \Delta T_{tt} \cdot \delta t \cdot \Delta A \cdot l_p)}{k_B T_{temperature}} \tag{1A}$$

And using Ng's infinite quantum statistics, we have to first approximation [24, 25]

$$\begin{aligned}
S(\text{entropy}) &\sim \ln Z + \frac{((E \sim \Delta T_{it}) \cdot \delta t \cdot \Delta A \cdot l_p)}{k_B T_{\text{temperature}}} \\
&\sim \ln Z + \left(\frac{h}{k_B T_{\text{temperature}} \delta g_{it}} \right) \\
&\xrightarrow{T_{\text{temperature}} \rightarrow \# \text{anything}} [S(\text{entropy}) \sim n_{\text{count}}] \neq 0 \\
&\quad (2A)
\end{aligned}$$

This is due to a very small but non vanishing δg_{it} with the partition functions covered by [23], and due to [24,25] with n_{count} a non-zero number of initial ‘particle’ or information states, about the Planck regime of space-time, so that the initial entropy is non-zero.

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